Note on a Toroidal Laser Resonator

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In several papers Röss and collaborators [1] have studied the properties of toroidal lasers with rectangular cut. For such a kind of laser resonator Itzkan [2] has calculated the total loss of the ray during one circuit in the limit of an ever increasing number of reflections. Till now, however, the modal properties of the field inside the resonator have very little been investigated. In this note we shall be engaged in the optical stability of the beam of rays and point out the advantage of the toroidal resonator with a circular cut.

Consider a resonator the reflection planes of which are determined by the surface of the rectangular toroid (Fig. 1). Inside the toroid, there is an active dielectric

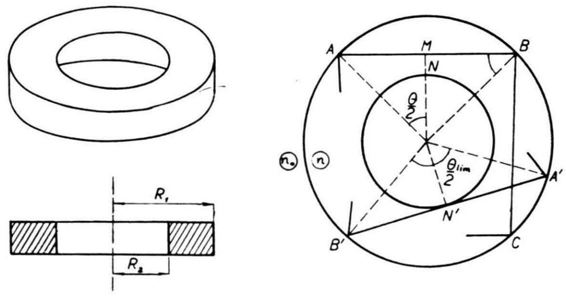

Fig. 1.

medium with a refractive index n, and the surroundings have a refractive index n_0. If the ray is to return into its initial point after one circuit, the central angle between two reflections must be $\Theta = 2\,\pi/m$, where m is the total number of reflections, i. e. the number of sides of the ray polygon. At each reflection the angle of incidence is $i = \pi/2 - \pi/m$. For $n_0 < n$, we make use of the total reflection, and by Snell's law the central angle Θ will satisfy the inequality

$$\Theta \leqq \Theta_{\text{lim}} = 2 \arccos(n_0/n).$$

The shortest distance of the ray from the centre of the toroid is $R_1 \cos(\Theta/2)$, where R_1 is the external radius of the meridional cut. Furthermore, the ray may not leave the inside of the toroid, and then $R_1 \cos(\Theta/2) = R_2$ holds, where R_2 is the internal radius of the toroid. Therefore, for a central angle of the ray polygon we have the condition

$$\Theta \leqq \min\left\{2 \arccos\left(\frac{n_0}{n}\right);\ 2 \arccos\left(\frac{R_2}{R_1}\right)\right\} \qquad (1)$$

For the critical angle ray (A′B′; Fig. 1) we obtain from the condition (1) the wellknown law [3]

$$R_1/R_2 = n/n_0\,. \qquad (2)$$

Let the ray (e. g. AB; Fig. 1) be the central ray of a narrow beam, and let us determine the stability of this beam. For the rectangular beam we can investigate the stabilities of both perpendicular longitudinal cuts separately. The general condition for stability of the optical resonator with two spherical mirrors is given by the inequality [4]

$$0 < \left(1 - \frac{L}{R'}\right)\left(1 - \frac{L}{R''}\right) < 1\,, \qquad (3)$$

where L is the length of the resonator, and R', R'' are the radii of the mirrors. In our case the mirrors are equal, and the condition (3) will be

$$0 < \frac{L}{f} < 4\,, \qquad (4)$$

where $f = R/2$ is the focal length of the mirrors.

In the longitudinal cut perpendicular to the plane of the toroid the beam is on the edge of stability (plane-parallel resonator), as for the $R = \infty$, $L/R = 0$. For the discussion of stability of the longitudinal cut parallel with the plane of the toroid we have to find the focus of the off-axis cylindrical mirror. From the geometrical optics of mirrors with oblique incidence we know [5], that in the meridional plane the focal lenght is

$$f_{\parallel} = (R/2)\cos i = (R/2)\sin(\Theta/2)\,.$$

When we put the focal length $f_{\parallel}$ and the length of the resonator $L = 2\,R_1 \sin(\Theta/2)$ into the relation (4), we obtain $L/f_{\parallel} = 4$. We see that also for the parallel cut, the beam is on the edge of stability (concentric resonator). Consequently, whispering modes only work out in this cavity.

If we want to increase the stability of the beam, we can change only the curvature in the cut perpendicular to plane of the toroid. We obtain the toroid with non-rectangular cut (Fig. 2). In this case, for the plane perpendicular to the meridional one the focal length is [5]

$$f_{\perp} = R_{\perp}/2 \cos i = R_{\perp}/2 \sin(\Theta/2)\,.$$

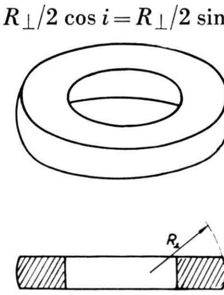

Fig. 2.

[1] For example: H. Hantsche and D. Röss, Z. Naturforschg. **18 a**, 1020 [1963].
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Thus, by applying the inequality (4) the condition of stability will be

$$0 < \frac{2\,R_1 \sin(\Theta/2)}{R_{\perp}/2 \sin(\Theta/2)} < 4\,, \tag{5}$$

where from

$$0 < \frac{R_1}{R_{\perp}} \sin^2(\Theta/2) < 1\,. \tag{6}$$

A special case of the circular toroid (Fig. 3), when $R_{\perp} = (R_1 - R_2)/2$, leads to the condition of stability

$$0 < \frac{2\,R_1}{R_1 - R_2} \sin^2(\Theta/2) < 1\,.$$

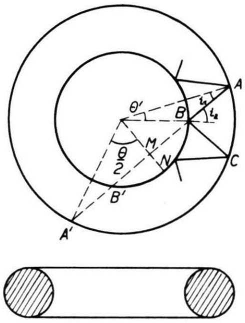

Fig. 3.

Reflections can also occur alternately at the external and internal surfaces of the toroid (Fig. 3). In such a case, the condition for the central angle will be

$$2 \arc\cos\left(\frac{R_2}{R_1}\right) \leqq \Theta \leqq 2 \arc\cos\left(\frac{n_0}{n}\right), \tag{7}$$

and the angle Θ' between two adjacent reflections (external and internal) is

$$\Theta' = \frac{\Theta}{2} - \arc\cos\left(\frac{R_1 \cos\Theta}{R_2}\right). \tag{8}$$

Let us investigate this beam in the meridional plane. From the geometry of the rays we conclude (Fig. 3) that a point M is the common meridional centre $2\,f_{\|} = R_{\|}$ of both surfaces, the external and internal respectively. In that case we obtain the condition of stability from the inequality (3)

$$0 < \left(1 - \frac{L'}{R_2 \cos i_2}\right)\left(1 - \frac{L'}{R_1 \cos i_1}\right) < 1 \tag{9}$$

which is hard to solve. For the limiting case of the central angle $\Theta' = 0$ one has

$$0 < \left(1 - \frac{R_1 - R_2}{R_1}\right)\left(1 - \frac{R_1 - R_2}{R_2}\right) < 1$$

leading to the inequality

$$0 < \frac{R_1 - R_2}{R_1} < \frac{1}{2}\,, \tag{10}$$

which is the condition of stability of the beam in the meridional plane (for $\Theta' = 0$).

In the plane perpendicular to the meridional one the condition of stability is

$$0 < \left(1 - \frac{2\,L' \cos i_1}{R_1 - R_2}\right)\left(1 - \frac{2\,L' \cos i_2}{R_1 - R_2}\right) < 1\,. \tag{11}$$

For the limiting case of the central angle $\Theta' = 0$ the resonator behaves as the concentric one, because $L' = R_1 - R_2$ and $\cos i_1 = \cos i_2 = 1$. As the angle Θ' increases, the resonator becomes more stable.

Beschleunigung von inhomogenen Plasmen durch Laserlicht

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Wenn das Licht hoher Intensität, wie es bei Riesenlaserpulsen entsteht, mit einem Plasma in Wechselwirkung tritt, so wird eine stark anisotrope Expansion des Plasmas beobachtet, die hauptsächlich dem Laserlicht entgegen gerichtet ist. So erhalten bei Einwirkung des Laserlichtes auf Festkörper im Vakuum die dem Laserlicht entgegenfliegenden Ionen Energien von 1 keV [1] und mehr [2]. Wenn Laserlicht in einem Gas fokussiert wird und dort einen Durchbruch erzeugt, zeigen von der Seite aufgenommene Schmierkamerabilder die Expansion dem Laserlicht entgegen. Die gemessenen Geschwindigkeiten ergeben Ionenenergien von derselben Größe wie bei Festkörpern [3]. Noch deutlicher wurde diese anisotrope Expansion des in Stickstoff erzeugten Plasmas durch die Messungen von Mandelstam u. a., nach denen das Intensitätsverhältnis der Linien und die Doppler-Verbreiterung des senkrecht gestreuten Lichtes eine Elektronentemperatur in der Größenordnung von 10 eV ergaben [4, 5], und eine Doppler-Verschiebung [4, 6] eine Expansion des Laserlichtes dem Licht entgegen zeigt, deren Geschwindigkeit einer Ionenenergie von einigen Hundert eV entspricht.

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